Instructor_Chapter11_VirtualWork (mfsilva@illinois.edu 2).pptx

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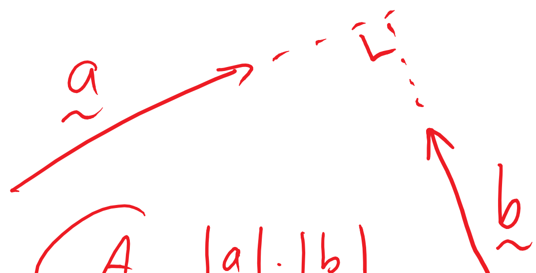
Chapter 11: Virtual Work

- An analytical tool to add to your skillset
- Can make complex frame & machine analysis simpler to perform

iClicker

Dot Products



$\underset{\sim}{a} \cdot \underset{\sim}{b} =$ ____

- A $|\underset{\sim}{a}| \cdot |\underset{\sim}{b}|$
- B $-|\underset{\sim}{a}| \cdot |\underset{\sim}{b}|$
- C 0
- D else

If $\underset{\sim}{a} = (1,5,3)$

$\underset{\sim}{a} \cdot \underset{\sim}{b} =$

- A 13
- B 11

$\underset{\sim}{a}$ & $\underset{\sim}{b} = (0, 2, 1)$ ... C. else

D. 0

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = 1 \times 0 + 5 \times 2 + 3 \times 1$$
$$= 0 + 10 + 3 = 13$$

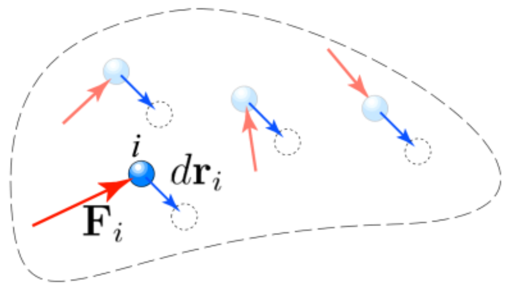
$\underset{\sim}{a} \cdot \underset{\sim}{b}$ is a
- A. scalar
- B. vector
- C. tensor

Definition of Work

Work of a force

A force does work when it undergoes a displacement in the direction of the line of action.

The work dU produced by the force $\boldsymbol{F}$ when it undergoes a differential displacement $\boldsymbol{dr}$ is given by



$$dU = \mathbf{F} \cdot d\mathbf{r}$$

Work is a scalar

- can be positive or negative
- has dimensions of force × length (energy)
- units: Joules (N·m)

(1 J = 1 N·m)

Key: dU is very different from torque (or moment)

What else has same dimensions as work?

A) $Q = \int y \, dA$

B) $I = \int y^2 dA$

C) $\underset{\sim}{M} = \underset{\sim}{r} \times \underset{\sim}{F}$

D) mass

{ torque, moment of a force, moment, couples }

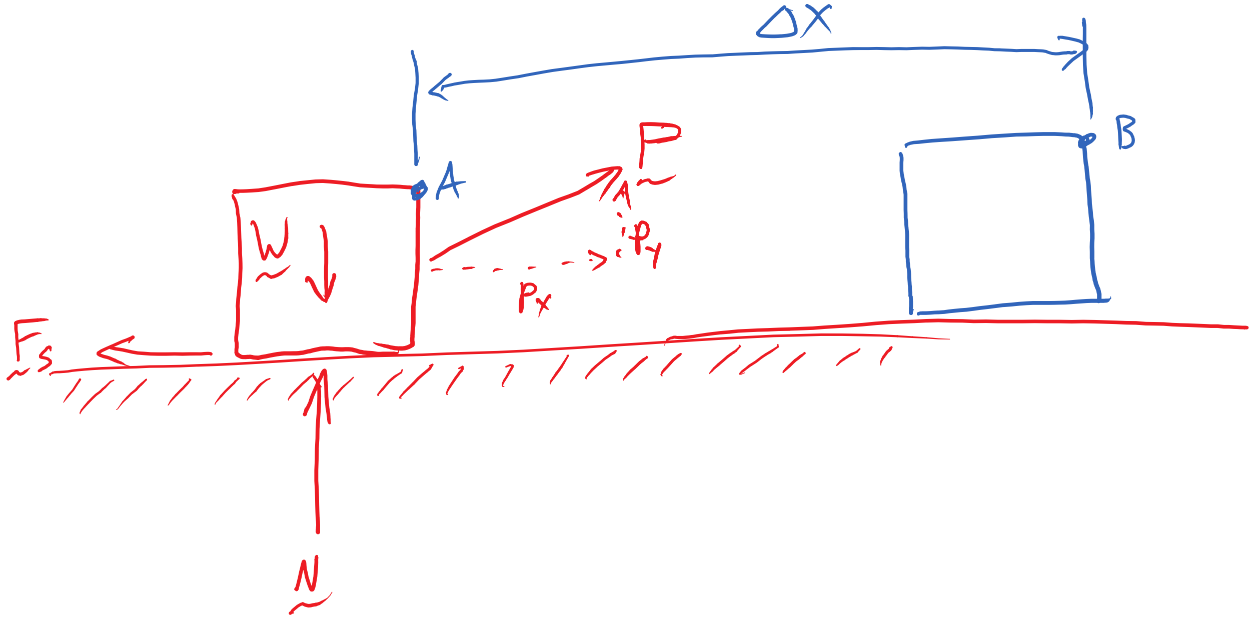
Can dU be added to $\underset{\sim}{M}$?

A) Yes

B) No

C) Sometimes

Moment $\underset{\sim}{M}$ is a vector (magnitude & direction)



$$\underset{\sim}{\Delta X} = \Delta X\,\hat{i}$$

Work done by each force

$$dU_P = \underset{\sim}{P} \cdot \underset{\sim}{\Delta X} = (P_x\hat{i} + P_y\hat{j}) \cdot \Delta X\,\hat{i} = P_x \cdot \Delta X$$

$$dU_W = \underset{\sim}{W} \cdot \underset{\sim}{\Delta X} = 0$$

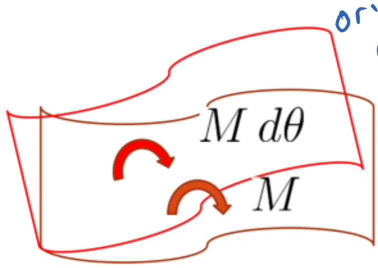
$$dU_N = \underset{\sim}{N} \cdot \underset{\sim}{\Delta X} = 0$$

$\underset{\sim}{W}$ & $\underset{\sim}{N}$ act $\perp$ to $\underset{\sim}{\Delta X}$
$\Rightarrow$ No work!

$$dU_{F_s} = \underset{\sim}{F_s} \cdot \underset{\sim}{\Delta X} = (-F_s\,\hat{i}) \cdot (\Delta X\,\hat{i}) = -F_s \cdot \Delta X$$

Definition of Work

Work of a moment $dU = M\mathbf{k} \cdot d\theta\,\mathbf{k} = M\,d\theta$



original orientation

final orientation

$\underset{\sim}{M}$ is not energy

$\underset{\sim}{M} \cdot \underset{\sim}{d\theta}$ is an energy quantity

dimensionless

dimensions force × length

$$dU = \underset{\sim}{M} \cdot \underset{\sim}{d\theta}$$
$$= (M\hat{k}) \cdot (d\theta \cdot \hat{k})$$
$$= M \cdot d\theta\,(\hat{k} \cdot \hat{k})$$
$$= M \cdot d\theta$$

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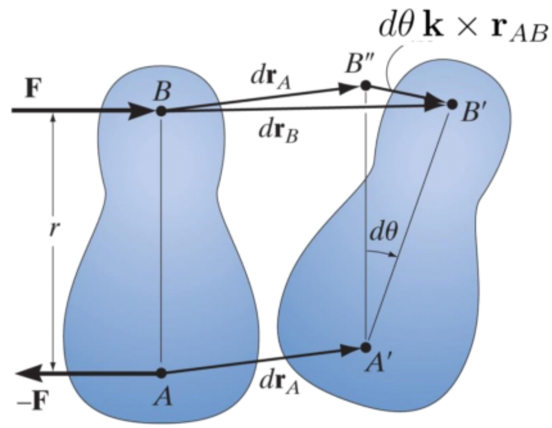
Definition of Work

Work of couple



$$d\mathbf{r}_P = d\mathbf{r}_A + d\theta\,\mathbf{k}\times\mathbf{r}_{AP}$$

$$\begin{aligned}
dU &= \sum_i \mathbf{F}_i\cdot d\mathbf{r}_i \\
&= \mathbf{F}_A\cdot d\mathbf{r}_A + \mathbf{F}_B\cdot d\mathbf{r}_B \\
&= -\mathbf{F}\cdot(d\mathbf{r}_A + d\theta\,\mathbf{k}\times\mathbf{r}_{AA}) + \mathbf{F}\cdot(d\mathbf{r}_A + d\theta\,\mathbf{k}\times\mathbf{r}_{AB}) \\
&= \mathbf{F}\cdot(d\theta\,\mathbf{k}\times\mathbf{r}_{AB}) \\
&= d\theta\,\mathbf{k}\cdot(\mathbf{r}_{AB}\times\mathbf{F}) \\
&= d\theta\,\mathbf{k}\cdot\mathbf{M}
\end{aligned}$$

The couple forces do no work during the translation $d\boldsymbol{r}_A$

Work due to rotation

Virtual Displacements

A *virtual displacement* is a conceptually possible displacement *or* rotation of all *or* part of a system of particles. The movement is assumed to be possible, but actually does not exist.

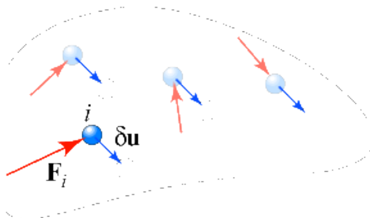
A virtual displacement is a first-order differential quantity denoted by the symbol δ (for example, δr and $\delta\theta$.

Principle of Virtual Work

The principle of virtual work states that if a body is in equilibrium, then the algebraic sum of the virtual work done by all the forces and couple moments acting on the body is zero for any virtual displacement of the body. Thus,

$$dU = \sum_i \underset{\sim}{F}_i \cdot \delta \underset{\sim}{u}_i + \sum_i \underset{\sim}{M}_i \cdot \delta \underset{\sim}{\theta}_i = 0$$

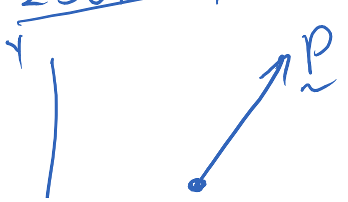
$$\delta U = \sum F\,\delta x + \sum M \cdot \delta\theta$$



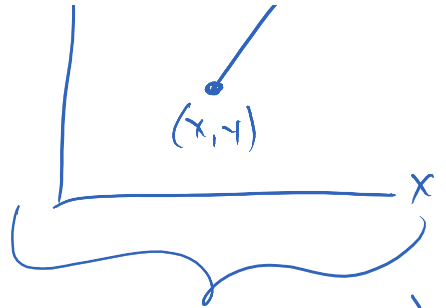
* useful for solving equilibrium problems of several connected rigid bodies
* Rigid ⟹ we do not consider internal forces/moments
* We consider single degree-of-freedom problems (1DOF)

Degrees of Freedom = # of unique variables required to describe a system's motion

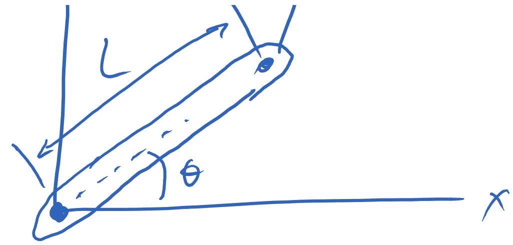
2DOF system



1DOF



requires (x,y) to describe where $\underset{\sim}{p}$ acts



Only θ is needed to describe where $\underset{\sim}{p}$ acts

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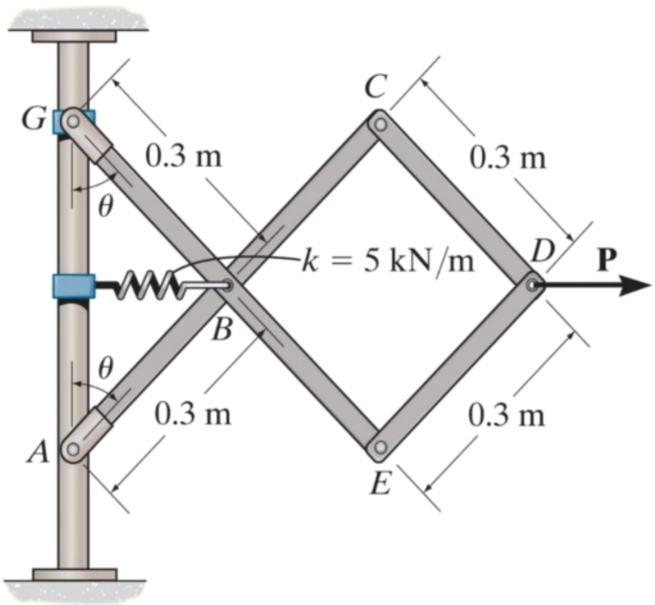
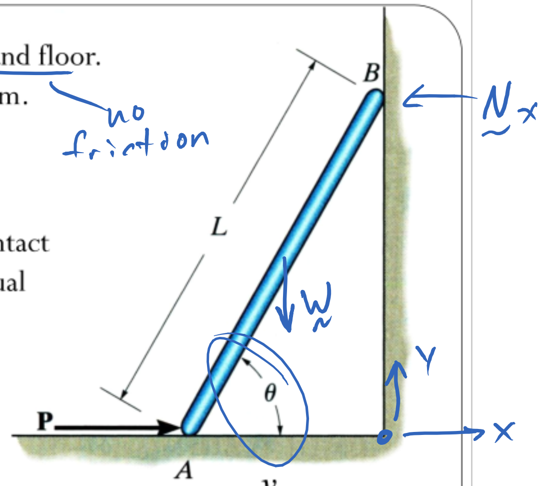


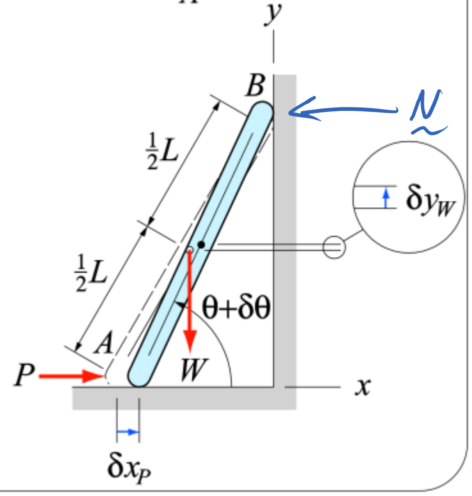
Determine the required force P needed to maintain equilibrium of the scissors linkage when the angle is 60 degrees. The spring is unstretched when the angle is 30 degrees.

Example: The thin rod of weight W rests against the smooth wall and floor. Determine the magnitude of force P needed to hold it in equilibrium.

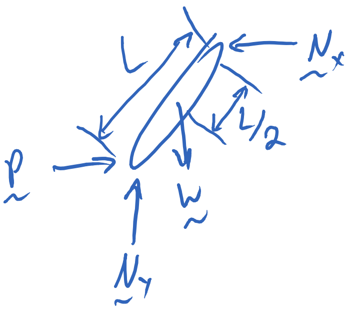
no friction



Use the principle of virtual work. This problem has one degree of freedom, which we can take as the angle θ. Let $\delta\theta$ be the virtual rotation of the rod, such that the rod slides at A and B. Since the contact at A and B are smooth, the only forces that do work during the virtual displacements are P and W. Then the virtual work becomes:



FBD



$\underset{\sim}{P} = P\,\hat{i}$

$\underset{\sim}{W} = -W\,\hat{j}$

Describe positions where $\underset{\sim}{P}$ & $\underset{\sim}{W}$ act.

$\underset{\sim}{P}$ acts at A: $\underset{\sim}{x}_A = -L \cdot \cos\theta\,\hat{i}$

$\underset{\sim}{W}$ acts at $\underset{\sim}{r}_w = -\frac{L}{2}\cos\theta \cdot \hat{i} + \frac{L}{2}\sin\theta \cdot \hat{j}$

$$\delta \underset{\sim}{x}_A = L \cdot \sin\theta \cdot \delta\theta \cdot \hat{i}$$

$$\delta \underset{\sim}{r}_w = \frac{L}{2}\sin\theta \cdot \delta\theta\,\hat{i} + \frac{L}{2}\cos\theta \cdot \delta\theta \cdot \hat{j}$$

Virtual Work

$$\delta U = \underset{\sim}{P} \cdot \delta \underset{\sim}{x}_A + \underset{\sim}{W} \cdot \delta \underset{\sim}{r}_w = 0$$

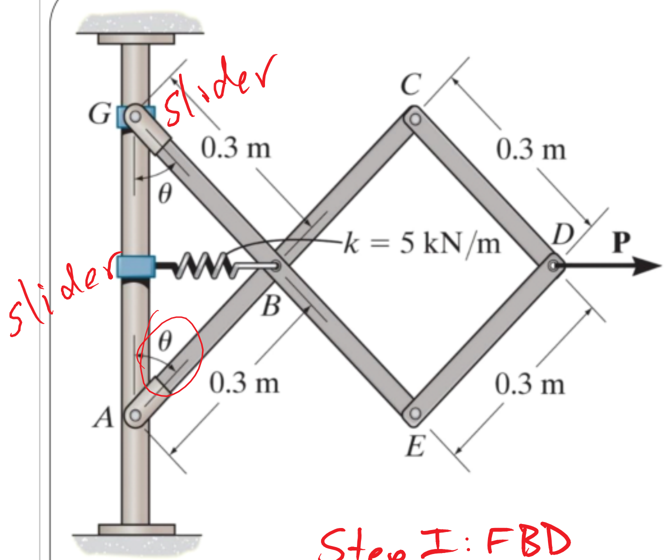
$$P\hat{i} \cdot L \cdot \sin\theta \cdot \delta\theta \, \hat{i} + (-W\hat{j}) \cdot \left(\frac{L}{2}\sin\theta \, \hat{i} + \frac{L}{2}\cos\theta \, \hat{j}\right) \cdot \delta\theta = 0$$

$$\underbrace{\left(P \cdot L \sin\theta - W \cdot \frac{L}{2}\cos\theta\right)}_{\text{must be zero}} \underbrace{\delta\theta}_{\text{"pretend" } \delta\theta \neq 0} = 0$$

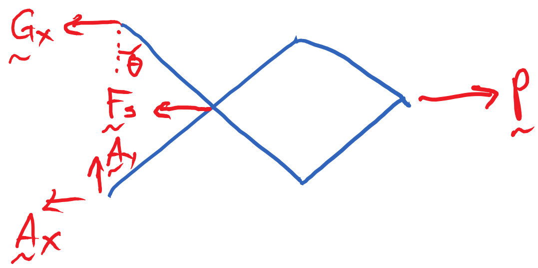
$$P \cdot L \cdot \sin\theta - W \cdot \frac{L}{2}\cos\theta = 0$$

$$\Rightarrow \boxed{P = \frac{W}{2 \cdot \tan\theta}}$$

Determine the required force P needed to maintain equilibrium of the scissors linkage when the angle is 60 degrees. The spring is unstretched when the angle is 30 degrees.



Step I: FBD



Step II: Virtual Displacements